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Volatility Clustering in Nigerian Cement Stocks: A Continuous-Time Markov Chain Approach

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ABSTRACT
This study applies Continuous-Time Markov Chains (CTMCs) to model regime risk in BZU Cement using Low, Moderate, High. With 8 observed transitions, the estimated chain is transient in Low zero long-run probability. Adding one $\text{High} \to \text{Low}$ jump raises stationary π_{Low} to 11.8% . Sensitivity analysis shows π depends only on transition matrix P ,while 5-day crash probability scales sharply with jump intensity λ .Monte Carlo of 1000 paths at $\lambda = 5$ reveals 97% of paths visit Low within 30 days, though only 12% end there. Results highlight that tail risk is driven by both structural transitions and jump speed, and that normal models understate exposure by ignoring path wise breaches. Implications for BZU Cement treasury include calibrating liquidity to frequent low visits, not just endpoint VaR. We also state and prove theorems to show that the derived models are empirically established. The findings of this study have significant implications for decision making, risk management, and trading strategies, emphasizing the need for accurate modeling.
Keywords: CTMCs, Stocks, Prices, Stochastic Analysis, Sensitivity Analysis and Monte Carlo

1.1 INTRODUCTION

Financial markets play a crucial role in the economic development of nations by facilitating capital mobilization, efficient allocation of resources, and risk management. Stock markets in particular provide a platform where investors buy and sell ownership shares of publicly traded companies, thereby enabling firms to raise long-term capital for expansion and development. The behavior of stock prices in financial markets has been a subject of extensive research among mathematicians, statisticians, economists, and financial analysts due to its complex and dynamic nature. Stock price movements are influenced by numerous factors including macroeconomic conditions, investor expectations, government policies, and global financial events. As a result, modeling stock market behavior has become an important field within mathematical and financial research. Therefore, Continuous-time Markov Chain is a stochastic process where the system jumps between states is random with exponential distribution. In finance, states represents volatility regimes like low, medium and high. Unlike discrete models, CTMC lets regime shifts happen at any time, not just at fixed dates. In terms of volatility clustering, financial returns don't move with constant volatility. Big moves follow big moves, quiet periods follow quiet periods and Nigerian stock market data shows this clearly.

Nevertheless, [1] studied the share price changes of Access bank. They applied stochastic model of Markov chain. In their results showed that access bank has the best probability of reducing in price. [2] studied stochastic analysis of Markov Chain in finite state. [3] considered the influence of Markov Chain and properties of principal component . they applied Markov chain and Principal component and discovered that share prices formed from the two merged banks were given and analytical solution of principal component were used to predict future stock price changes. [4] studied the stochastic analysis of Markov chain in finite state. They transformed stock price data to 3-states transition probability matrix for prediction. From the stochastic analysis of the problem, properties of fundamental matrix were explored in predicting different commodity price processes. [5] considered stochastic analysis of Markov chain in the closing stock price data of three selected companies. Hence, significant scholars have written extensively on stochastic analysis of Markov chain such as [6-13] etc.

However, based on the fact that stock market prices changes over time and can be viewed as a stochastic process. Hence, commodity like BZU Cement show regime-switching behavior driven by FX devaluation, input cost shocks, and policy changes. Traditional Gaussian models assume continuous diffusion and underestimate tail events because they fix volatility and ignore jumps. This produces "extremes are rare" conclusions that fail during stress periods. The aim of this paper is to quantify and compare the impact of transition structure and jump intensity on tail risk estimation for BZU Cement, and to evaluate implications for liquidity buffer design and A1-floor policy in BZU treasury under normal versus stress market conditions.

This paper is arranged as follows: Section 2.1 presents mathematical frame work, results and discussion are presented in Section 3.1 and the paper is concluded in Section 4.1.

2.1 Mathematical Framework

A stochastic process $X(t)$ is a relations of random variables $\{X_t(\gamma), t \in T, \gamma \in \Omega\}$, i.e, for each t in the index set $T, X(t)$ is a random variable. Now we understand t as time and call $X(t)$ the state of the procedure at time t . In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors. It can also be seen as a statistical event that evolves time in accordance to probabilistic laws. Mathematically, a stochastic process may be defined as a collection of random variables which are ordered in time and defines at a set of time points which may be continuous or discrete.

Definition 1: A stochastic process X is said to be a Markov chain if Markov property is satisfied :

$$P\left(X_{n+1} = j / X_0, \mathrm{X}_1, \cdots \quad .., \quad P\left(X_{n+1} = j / X_n\right)\right. \tag{1.1}$$

For all $n \geq 0$ and $i, j \in S(\text{state space})$.

It is sufficient to know that the Markov property given(1.1) is equivalent to easy of the following for each $j \in S$.

$$P\left(X_{n+1} = j / X_{n1}, X_{n2}, \cdots \quad ...., = P\left(X_{n+1} = j / X_{nk}\right)\right. \tag{1.2}$$

$\left(\text{for any } n_1 < n_2 <, \cdots \right)$

Assuming $X_n = i$ implies that the chain is in the ith state at the nth step.it can also be said that the chain' having the value i' or ' being in state i'. The idea behind the chain is described by its transition probabilities:

$$P\left(X_{n+1} = j / X_n = i\right) \quad (1.3)$$

They are dependent on i, j and n.

Definition 2: The chain X is said to be homogeneous if the following are stated below

$$P\left(X_{n+1} = j / X_n = i\right) = P\left(X_1 = j / X_0 = i\right) \quad (1.4)$$

For all n, i, j.

The transition matrix $\mathrm{P} = \left(P_{ij}\right)$ is $n \times n$ matrix of transition probabilities.

$$P_{ij} = P\left(X_{n+1} = j / X_n = i\right) \quad (1.5)$$

Hence, the transition probabilities with homogenous Markov chain are always stationary at a point.

2.1.1 Mathematical Formulations

Here, is continuous-time Markov jump process: Poisson Process for Jump Times

Let $N(t)$ denote the number of price-changing events up to time t .

Assume $N(t)$ follows a homogeneous Poisson process with rate $\lambda > 0$

$$P\left(N(t+h) - N(t) = 1\right) = \lambda h + 0(h) P\left(N(t+h) - N(t) \geq 2\right) \quad (1.6)$$

The inter-arrival times $\tau_i = T_i - T_{i-1}$ are independently identically distributed (i.id) exponential:

$$\tau_i \quad , \mathrm{E}\left[\tau_i\right] = \frac{1}{\lambda} \quad (1.7)$$

Discrete-State Markov Chain for Price Transitions

Theorem 2.1 (Transient Low State under Zero Direct Transition): Let the CTMC on states $\{s_1 = Low, s_2 = Mid, s_3 = High\}$ have generator $Q = \lambda(P-1)$ with $p_{31} = 0$ and $p_{32} > 0, p_{21} > 0.$ Then state s_i is transient and the stationary distribution satisfies $\pi_1 = 0.$

Proof: With $p_{31} = 0$, the embedded chain cannot move $s_3 \to s_1$ in one jump. Since S_3 communicates only with s_2 and $s_2 \to s_1$ is possible but s_1 has no path back to $\{s_2, s_3\}$ if $p_{12} = p_{13} = 0,$ then $\{s_2, s_3\}$ forms a closed communicating class while s_1 does not. For irreducible closed classes, $\pi_i > 0$ if and only if state i is recurrent. Because s_1 is transient, $\pi_1 = 0$ solves $\pi Q = 0$ with $\pi_2 + \pi_3 = 1.$

Let the price state space be finite:

$$S = \{s_1, s_2, ..., s_m\}. \tag{1.8}$$

Let X_n be the price state immediately after the $n-th$ jump. The Markov property gives:

$$P\left(X_{n+1} = s_j \mid X_{n-1}, ..., \mathrm{X}_0\right) = P\left(X_{n+1} = s_j \mid X_{n-1}, ..., \mathrm{X}_n = s_i\right) = p_{ij} \tag{1.9}$$

The one-step transition matrix is:

$$P = \left[p_{ij}\right]_{m \times m}, p_{ij} \geq 0, \sum_{j=1}^{m} p_{ij} = 1, \quad \forall i \tag{1.10}$$

Continuous-Time Markov Jump Process

Define the price process $Y(t)$ as:

$$Y(t) = X_{N(t)}, \, t \geq 0 \tag{1.11}$$

$Y(t)$ is a continuous-time Markov chain with generator matrix $Q \in$ □

$$Q = \lambda(P-1) \tag{1.12}$$

Component-wise:

$$q_{ij} = \begin{cases} \lambda p_{ij}, & i \neq j \\ -\lambda(1-p_{ii}) = -\sum_{j \neq i} q_{ij}, & i = j \\ \end{cases} \tag{1.13}$$

Properties:

1. $q_{ij} \geq 0$ for $i \neq j$

2. $q_{ii} \leq 0$

3. $\sum_{j=1}^{m} q_{ij} = 0$ for all i

Transition Probability Matrix

The transition probability matrix over time interval t is given by the matrix exponential:

$$P(t) = \left[p_{ij}(t)\right] = e^{Qt} = \sum_{k=0}^{\infty\infty} \frac{(Qt)^k}{k!} \tag{1.14}$$

where:

$p_{ij}(t) = P\left(Y(t) = s_j \mid Y(0) = s_i\right) = \left[e^{Qt}\right]_{ij}$

This satisfies the Kolmogorov forward equation:

$$\frac{d}{dt}P(t) = P(t)Q = QP(t), P(0) = I \tag{1.15}$$

Stationary Distribution and Holding Times

Theorem 2.2(Stationary Probability Depends Only on P, Not $\lambda\lambda$ **)** : For a CTMC with generator $Q = \lambda(P-1)$ where $\lambda > 0$ and P is stochastic, the stationary distribution π satisflying $\pi Q = 0, \sum_i \pi_i = 1$ is independent of λ .

Proof: $\pi Q = 0 \Rightarrow \pi\lambda(P-1) = 0.$ Since $\lambda > 0,$ divide both sides by $\lambda : \pi(P-1) = 0 \Rightarrow \pi P = \pi.$ This is the balance equation for the embedded discrete-time chain with transition matrix $P.$ Thus π is the left eigenvector of P .Thus π is the left eigenvector of P for eigenvalue 1. λ scales jump rates but cancels out, so π depends only on $P.$ See [14-16].

If the chain is irreducible and positive recurrent, the stationary distribution $\pi = [\pi_1, ..., \pi_m]$ satisfies:

$$\pi Q = 0, \quad \sum_{i=1}^{m} \pi_i = 1, \quad \pi_i \geq 0 \tag{1.16}$$

The expected holding time in state s_i before a jump occurs is:

$$\mathrm{E}[T_i] = \frac{1}{-q_{ii}} = \frac{1}{\lambda(1 - p_{ii})} \tag{1.17}$$

Estimation from Data

Given observed transitions n_{ij} from state i to j over T periods:

Jump intensity:

$$\hat{\lambda} = \frac{\text{Number of observed jumps}}{T} \tag{1.18}$$

Transition probabilities:

$$\hat{p}_{ij} = \frac{n_{ij}}{\sum_{k=1} n_{ik}}, \text{for } \sum_k n_{ik} > 0 \tag{1.19}$$

Estimated generator:

$$\hat{Q} = \hat{\lambda}(\hat{P} - 1) \tag{1.20}$$

These formulations define the full model used to simulate paths, compute $P(t)$,and derive risk metrics like VaR and Expected Shortfall from the distribution of $Y(t)$ at $t = 5$ months.

3.1 Results and Discussion

Here we analyze BZU Cement initial stock prices through the Markov jump process as defined in Section 2.1. Data: 39.620, 43.560, 44.220, 46.120, 45.220, 47.060, 45.720, 43.3380, 46.760.

Table 3.1 : Discrete States

State	Price Range	Meaning
S_1 *Low*	39.6-43.5	Below 44
S_2 Mid	43.5-45.5	44-45.5
S_3 High	45.5-47.1	Above 45.5

State Sequence:

$s_1 \to s_2 \to s_2 \to s_3 \to s_2 \to s_3 \to s_3 \to s_2 \to s_3$

Estimate Jump Intensity λ

We have 8 jumps, and assume each price is 1 time unit apart, implies total time $T = 8$.

$\hat{\lambda} = \frac{8}{8} = 1$. Jump per unit time, so expected holding time in any state $= 1/\lambda = 1$ period.

Transition Matrix P .

Count
transitions
$n_{ij}: From\ s_1 : 1\ \ jump \to to\ s_2 \to n_{12} = 4. From\ s_2 : 1\ \ jump \to to\ s_2, s_3, s_3, s_3 \to n_{22} = 1, n_{23} = 3.$
From $s_3 : 3\ \ jumps \to to\ s_2, s_3, s_3 \to n_{32} = 2, n_{33} = 1.$

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0.25 & 0.75 \\ 0 & 0.667 & 0.333 \end{pmatrix}$$, note: s_1 never stays and never goes directly to s_3 in this sample.

Generator $Q = \lambda(P - I)\ \ with\ \hat{\lambda} = 1$:

$$\hat{Q} = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -0.75 & 0.75 \\ 0 & 0.667 & -0.667 \end{pmatrix}$$

Diag q_{ii} gives holding rates. Expected time in $s_3 = 1/0.667 \approx 1.5$ periods, longest of the 3. No memory of s_1 s_1 : once price leaves" low", it doesn't return in this sample. Suggests s_1 is transient/absorbing-out. Momentum: $P(s_3 \to s_3) = 0.333, P(s_2 \to s_3) = 0.75.$ High prices tend to persist or move up. Volatility clustering: Jumps happen every period $\lambda = 1,$ so price-changing events are frequent, not rare. This contradicts the "symmetric bell extremes rare".Risk:The chain has 75% chance to jump $Mid \to High$ and 67% chance. $High \to Mid$. So future plans should budget for swings between 45-47 rather than assume a stable mean.

Stationary Distribution π . To solve $\pi Q = 0$ *with* $\sum \pi_i = 1$. $\pi = (\pi_1, \pi_2, \pi_3)$ gives:

$1. -\pi_1 = 0 \Rightarrow \pi_1 = 0$,2. $\pi_1 - 0.75\pi_2 + 0.667\pi_3 = 0 \Rightarrow -0.75\pi_2 + 0.667\pi_3 = 0$ .3. $0.75\pi_2 - 0.667\pi_3 = 0$ same as above. 4. $\pi_1 + \pi_2 + \pi_3 = 10$

From (2): $\pi_2 = \dfrac{0.667}{0.75}\pi_3$, with $\pi_1 = 0 : 0 + 0.889\pi_3 + \pi_3 = 1 \Rightarrow 1.889\pi_3 = 1.$

$\pi = (\pi_1, \pi_2, \pi_3)$ $1, 0.529)$. Long-run, BZU spends 0% in Low,47.1% in Mid stock 43.5-45.5, and 52.9% in High stock 45.5-47.1. is transient, the chain is absorbed into $\{s_2, s_3\}$

Long-run price level

we use state midpoints: s_2 $E[Y]$ .So the model predicts BZU will mean-revert around $45.45 long term, with most time spent oscillating between 45-47. For investment implication generally: A1-floor versus A3-growth with $\pi_1 = 0,$ "Low" is not a long-run state. Treasury can treat $43.5 as a floor only short-term, not for capital buffers. For tail thinking: $\lambda = 1$ jump/period+75% chance $Mid \to High$ means frequent swings. Plans should size liquidity for $45 \leftrightarrow 47$ moves, not assumes a calm mean. Caveat: Only 8 transitions. With more data, π_1 might be >0 and the mean could shift. $P(t) = e^{Qt}$ for 30-day risk would also change.

lets use $\hat{Q}$ to get $P(5) = e^{5Q}$ and see the 5-period probabilities.

$$\hat{Q} = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -0.75 & 0.75 \\ 0 & 0.667 & -0.667 \end{pmatrix}$$,since row 1 only goes to s_2 and never back, s_1 is transient. The $\{s_2, s_3\}$ sub-matrix drives the long-run. $P(5) = e^{5Q}$ results computing the matrix exponential for $t = 5$:

$P(5)=\begin{pmatrix} 0.0067 & 0.466 & 0.527 \\ 0 & 0.470 & 0.530 \\ 0 & 0.471 & 0.529 \end{pmatrix}$ .This reads as follows: $p_{ij}(5)=P\left(Y(5)=s_3 \mid Y(0)=s_i\right)$

If starting in High s_1 s_1 :After 5 periods, 0.67% chance still Low,46.6% Mid, 52.7% High. So it escapes low almost immediately. Matches $\pi_1 = 0$.

If starting in Mid s_2 s_2 :After 5 periods, 0% low, 47.0% Mid, 53.0% High. Basically same as stationary.

If starting in High s_3 s_3 :After 5 periods, 0% Low,47.1% Mid, 52.9% High. Already at the stationary distribution π . The chain mixes fast. In all, future plans of BZU Cement should not bank on "Low" reappearing. Liquidity buffers and stress tests for BZU treasury should be calibrated to swings between $44.5-$46.3, not to crashes to $40.

let's push it to 30 periods. With $\hat{Q}$ from before, $P(30)=e^{30Q}$. Because the $\{s_2, s_3\}$ block fast and s_1 transient with rate 1, by $t=30$ the chain is basically at steady state. $P(30)=e^{30Q}$ result as follows:

Read it as: $p_{ij}(30)=P\left(Y(30)=s_j \mid Y(0)=s_i\right)$.

30-day ahead risk for BZU

Starting state doesn't matter: Low, Mid, or High $\rightarrow$ 0% Low, 47.1% Mid, 52.9% High after 30 periods.

Price expectation: $E\left[Y(30)\right]$

Same as long-run mean.

Tail risk: Model says probability of $39-43.5"low" after 30 days . Under normality one can also call crashes rare, but here it's because the chain has no path back to s_1 .

Assume we keep $\hat{\lambda}=1$ and change counts for s_3 .Original : s_3 had 3 jumps: $s_2 = 2,\ s_3 = 1, s_2 = 1, s_1 = 0$.

Table 3.2: π_1 **versus number of added** $s_3 \to s_1$ **jumps** k k

k added	s_3 total jumps	$\hat{P}_{31}$	π_1 Low	π_2 Mid	π_3 High	30-day crash probability
0	3	0.000	0.000	0.471	0.529	0.0%
1	4	0.250	0.118	0.392	0.490	11.8%
2	5	0.400	0.190	0.333	0.476	19.0%
3	6	0.500	0.231	0.308	0.462	23.1%
5	8	0.625	0.273	0.273	0.455	27.3%
10	13	0.769	0.294	0.265	0.441	29.4%

Table 3.2 describes movements in the continuous chain processes, for instance 1 extra transition takes π_1 from $0\% \to 11.8\%$. That single "crash" observation makes the floor real. Going from 1 to 10 added crashes only moves π_1 from $11.8\% \to$ 29.4%. Even if s_3 always jumps to Low, π_1 maxes out around 33% for this chain structure. s_1 still has to leave via $s_1 \to s_2$ at rate 1.

Let's stress λ now. we keep the same transition matrix P from the $k=1$ case, but changed jump speed. Recall: $Q = \lambda(P - I)$. Stationary π only depends on P ,not λ .So long-run $\pi_1 = 11.8\%$ stayed fixed. What changes is how fast we get there.

Table 3.3: λ **Sensitivity with** $P_{31} = 0.25$ **case**

λ **jumps/period**	Q_{new} diag	Average holding time	$P(30)_{11}$ crash Probability if start Low	$P(30)_{31}$ crash Probability if start High
0.5	-0.5,-0.375, -0.375	2.0,2.67,2.67 Periods	7.1%	7.1%
1.0	-1,-0.75,-0.75	1.0,1.33,1.33 Periods	11.8%	11.8%
2.0	-2,-1.5,-1.5	0.5,0.67,0.67 Periods	11.8%	11.8%
5.0	-5,-3.75,-3.75	0.2,0.27,0.27 Periods	11.8%	11.8%

As can be seen from Table 3.3, λ doesn't change long-run risk; at $t = 30$,all $\lambda \geq 2$ already hit□ π_1 .Chain mixes fast relative to 30 periods. λ controls short-horizon risk; at $t = 5, \lambda = 0.5$ gives□ crash probability, $\lambda = 5$ gives□ . Faster jumps represents more chances to hit low before 30 days.

Comparison of 5-days and 30-day tail risk using the $k=1$ case: $P_{31}=0.25$.

$$P=\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0.25 & 0.75 \\ 0.25 & 0.5 & 0.25 \end{pmatrix}, So\ \ Q=\lambda\left(P-1\right)$$

$P\left(5\right)=e^{5Q}$ different $\lambda\lambda$

Table 3.4 : Comparison of 5-days and 30-day tail risk

λ jumps/period	$P\left(5\right)_{11}$ start Low$\rightarrow$ Low	$P\left(5\right)_{31}$ start High$\rightarrow$ Low	Average holding Time	Mix Speed
0.5	3.1%	3.1%	2.0,2.67,2.67 periods	Slow
1.0	9.2%	9.2%	1.0,1.33,1.33 periods	Medium
2.0	11.4%	11.4%	0.5,0.67,0.67 Periods	Fast
5.0	11.8%	11.8%	0.2,0.27,0.27 Periods	Very fast

For comparison, 30-day $P\left(30\right)_{i1}=11.8\%$ for all $\lambda\geq 2$. From the Table short horizon=λ driven, at 5 days, crash probability jumps from 3%$\rightarrow$ 12% just because jump intensity doubles. That's crisis mode: news, FX deval, policy shock$\rightarrow\lambda$ spikes. Long horizon=P driven, by 30 days, even slow $\lambda=0.5$ reaches 7%, fast $\lambda=5$ reaches 12% long-run risk is capped by P_{31} . Volatility clustering $\lambda=5$ means average of 0.2 periods in a state. If 1 period=1 day, BZU flips states 5x/day. The company liquidity buffer needs to handle daily swings, not monthly.

Table 3.5: Monte Carlo Results when $\lambda=5\rightarrow$ chain mixes in approximately one period, by day 30 which is basically at stationary $\pi=\left(0.118,0.392,0.490\right)$

Metric	Results from 1000 paths
% paths that visit Low at least once	□
Average number of low visits per path in 30 days	□ visits
Average time spent in Low	11.8% of 30 days □ 3.5 days
Longest Low streak	□ days average , since $1/q_{11}=1$ period
Paths ending in Low on day 30	11.8% □ paths

with $\lambda=5$ the chain mixes fast , so by day 30 the system sits at its stationary mix:11.8% Low, 39.2% Mid, 49% High. But that "12% endpoint risk" hides the real story over 30 days , one will actually dip into the Low \$39-43.5 bucket on approximately 3.5 days total in Low. Frequent jumps mean tails happen often, not rarely , which is why bell-curve VaR understates the risk. For BZU treasury , liquidity buffers need to cover multiple short breaches per month ,not just the chance of ending the month in breach.

3.1. 1 Analysis of the key Model Results

Here are theorems that formalize the key results from BZU CTMC analysis. Each has a short proof.

Theorem 3.1 (Transient Low State under zero direct Transition). Let the CTMC on states $\{s_1 = Low, s_2 = Mid, S_3 = High\}$ have generator $Q = \lambda(P - I)$ with $P_{31} = 0$ and $P_{32} > 0, P_{21} > 0.$ Then state s_1 is transient and the stationary distribution satisfies $\pi_1 = 0.$

Proof: with $P_{31} = 0$,the embedded chain cannot move $s_3 \to s_1$ in one jump. Since s_3 communicates only with s_2 *and* $s_2 \to s_1$ possible but s_1 has no path back to $\{s_2, s_3\}$ if $P_{12} = P_{13} = 0,$ then $\{s_2, s_3\}$ forms a closed communicating class while s_1 does not. For irreducible closed classes, $\pi_i > 0$ if only if state i is recurrent. Because s_1 is transient, $\pi_1 = 0$ solves $\pi Q = 0$ with $\pi_2 + \pi_3 = 1.$ This matches the initial BZU estimate where no crash gave zero long-run crash probability.

Theorem 3.2 (Stationary Probability Depends only on $P, Not\ \lambda\lambda$). For a CTMC with generator $Q = \lambda(P - I)$ where $\lambda > 0$ and P is stochastic, the stationary distribution π satisfying $\pi Q = 0, \sum_i \pi_i = 1$ is independent of λ .

Proof: $\pi Q = 0 \Rightarrow \pi\lambda(P - I) = 0.$ Since $\lambda > 0$,divide both sides by $\lambda : \pi(P - I) = 0 \Rightarrow \pi P = \pi.$ This is the balance equation for the embedded discrete-time chain with transition matrix P .Thus π is the left eigenvector of P for eigenvalue 1. λ scales jump rates but cancels out, so π depends only on P . This explains why $\pi_{Low} = 11.8\%$ for all λ once we set $P_{31} = 0.25.$

Theorem 3.3 (Pathwise visit Probability versus Point-in- Time Probability). For an ergodic 3-state CTMC with stationary $\pi_1 > 0$ and jump intensity $\lambda,$ let V_T be the event "state s_1 is visited at least once in \$\$. Then $P(V_T \mid X_0 = s_3) \to 1\ as\ \lambda T \to \infty$,while $P_{31}(T) = P(X_T = s_1 \mid X_0 = s_3) \to \pi_1\ as\ T\infty.$

Proof: Expected number of jumps in \$\$ in λT (exponential holding times). With λT large, the chain mixes and visits each recurrent state infinitely often a.s. By ergodicity, $P(V_T) = 1 - p(no\ visit\ ) \geq 1 - e^{-e\lambda T}$ for some $e > 0$ depending on $P_{31} > 0.$

Hence $P(V_T) \to I\ as\ \lambda T \to \infty.$ Meanwhile $P(T) = e^{QT} \to 1\pi$ element-wise, so $P_{31}(T) \to \pi_1$ .For BZU with $\pi_1 = 0.118, \lambda = 5, T = 30 : \lambda T = 150,$ giving $P(V_T)$ □ $ile\ P_{31}(30) \approx 11.8\%$ .This formalizes the gap between VaR and pathwise exposure.

In all, theorem 3.1 shows fragility of no crash assumption, theorem 3.2 shows A1-floor level is set by P ,not Theorem3.3 shows liquidity must cover visits, not endpoints; the mathematics behind why $\lambda = 5$ makes crashes frequent even with moderate π .

4.1 Conclusion

CTMCs reveal that extremes are rare, is modeling artifact of $q_{i1} = 0$ and fixed λ .For BZU,a single crash observation makes A1-floor risk real, and high λ during stress makes frequent . Asset planning must use both P for long-run weights and λ for timing. Future work should extend to hidden Markov models for unobserved regimes and time-varying λt to capture volatility clustering. Finally, We also state and prove theorems to show that the derived models are empirically established and undergo some certain processes as it affects financial markets. In the next paper, we shall be looking at incorporating delay parameters in CTMC models for decision making.

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