

Experimental Investigation of Energy Dissipation in Hydraulic Jump: A Comparison of Weir and Level Bedded Constricted Flume

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Abstract

Hydraulic jumps are common occurrences in open channels due to transition of flows from supercritical flow to a subcritical or tranquil one. A flow transition will occur when there is a change in the channel depth or width. As a result of flow transition in an open channel with flowing liquid, a lot of energy of the flowing liquid is dissipated mostly in the form of heat (Tran, 2011). In most open channels with flowing liquid like rivers, dams and spillways, etc., the hydraulic jumps found are similar to standing waves that can be used as chemical mixers or pollution control aerators because of the considerable amount of air emission that accompanies the jumps (Jalil Sarhan and Yaseen, 2015). This paper experimentally investigates energy dissipation in hydraulic jump (a comparison of weir and level-bedded constricted flume). The experiments were carried out on the horizontal open channel provided in the fluid mechanics/hydraulic laboratory of the Cross River University of Technology, Calabar-Nigeria. On the whole, the experiment results show that there was flow continuity since the discharge through any point of the channel is approximately equal. In an open channel hydraulic jump actually occurs only when there is flow continuity and when a flowing liquid transits from supercritical flow to subcritical one. Hydraulic jump resulting from a weir dissipates more energy than that caused by a level-bedded constricted flume. The inflow Froude number F_r of a hydraulic jump is a function of sequent depth ratio of the post and pre-hydraulic jump sections or velocity ratio of pre and post-hydraulic jump sections, irrespective of the jump causative agent.

Keywords: Open Channel, Hydraulic jump, Froude Number, Weir, Flume, Specific Energy.

I. INTRODUCTION

According to Li (1995), the phenomenon of hydraulic jump began since 19th century. Sequel to that, in the early 80s Belanger had used *momentum principle* to derive solutions for hydraulic jump which was backed by Gibson (1913) verification experiments. Belanger proposed that a hydraulic jump will occur in a smooth rectangular open channel whenever sequent depth ratio or velocity ratio

$\frac{y_3}{y_2} = \frac{v_2}{v_3} = \frac{1}{2} \left(\sqrt{1 + 8F_{r2}^2} \right) - 1$ is satisfied between the pre and post-hydraulic jump sections, irrespective of the jump causative agent. This implies that hydraulic jump will occur in a smooth rectangular channel if the initial depth y_2 , the sequent depth y_3 and the inflow Froude number F_{r2} satisfy the above given momentum equation. The flow regime can be defined in terms of Froude number, F , by comparing the unit inertial force to the unit gravitational force (Forester and Skrinde, 1949; Leutheusser & Birk, 1991; and Leutheusser and Fan, 2001). The inflow Froude number F_{r2} is given as:

$$F_{r2} = \frac{V_2}{\sqrt{gy_2}}$$

When $F_{r2} > 1$ Supercritical flow occurs; when $F_{r2} = 1$ critical flow and $y_2 = y_3$; and when $F_{r2} < 1$ subcritical flow.

The overflow discharge per unit width can be obtained from:

$$q = \frac{2}{3} C_d \sqrt{2gH}^{3/2}$$

Hydraulic jumps are common occurrences in open channels due to transition of flows from supercritical flow to a subcritical or tranquil one. A flow transition will occur when there is a change in the channel depth or width. As a result of flow transition in an open channel with flowing liquid, a lot of energy of the flowing liquid is dissipated mostly in the form of heat. In most open channels with flowing liquid like rivers, dams and spillways, etc., the hydraulic jumps found are similar to standing waves that can be used as chemical mixers or pollution control aerators because of the considerable amount of air emission that accompanies the jumps. In this paper, the characteristics and energy dissipation in hydraulic jumps generated by a weir and a level-bedded constricted flume is investigated through physical experiments.

II. RELATED LITERATURE

A. Submerged Hydraulic Jump

As it is common with low low-head dams, when the tail water y_3 raises higher than the ideal sequent depth to satisfy the momentum equation $\frac{y_3}{y_2} =$

$\frac{v_2}{v_3} = \frac{1}{2}(\sqrt{(1 + 8F_{r2}^2)} - 1)$ condition, a submerged hydraulic jump which sweeps back on itself and creates a vortex will occur. The resulting vortex in turn swirls on a horizontal axis to create a strong pre-hydraulic jump surface velocity that pushes whatever it comes in contact with back into the dam (Leutheusser and Fan, 2001).

B. Importance of Hydraulic Jump

The importance of hydraulic jumps cannot over-emphasized. They act as energy dissipaters, aerators, etc. Hydraulic jumps influence the structural behaviours of most hydraulic structures such as dams, rivers, and spillways, etc.

C. Uses of Hydraulic Jump

The resulting vortex characteristics developed due to hydraulic jump can be used for chemical mixing activities. to maintain the high-water profile for irrigation and supplies. It can be used to prevent scouring and erosion of hydraulic structures. It can be used water storage, regulation and supplies.

III. METHOD

A. Experimental Work

1) Specimens

a. Weir (25 x 75 x 114mm steel tablet)

To create a 0.075m wide, 0.114m long 0.075m high broad crested weir, three replaceable tablets were placed 2.5m away from the channel outlet. The weir creates and maintains the upstream supercritical flow conditions required for the experiments. Froude number obtained during the experiments varied from 0.068 to 3.56.



Fig. 1: 25 x 75 x 114mm steel tablet



Fig. 2: Sample Weir and Hydraulic Jump

b. Flume Slides

2 rectangular plexiglass slides, tapered at both ends. Each glass is (12.5mm) thick, 0.15m (150mm) high and 0.34m (340mm) long. During the experiment, a level-bedded constricted flume with a 0.50mm (50mm) throat was created by sandwiching the 2 slides to the opposite walls 2.5m away from the channel outlet. The channel constriction causes and maintains the upstream rapid flow conditions required for the jump experiments. The Froude number obtained during the experiments varied from 0.38 to 1.93.



Fig. 3: Flume Slides



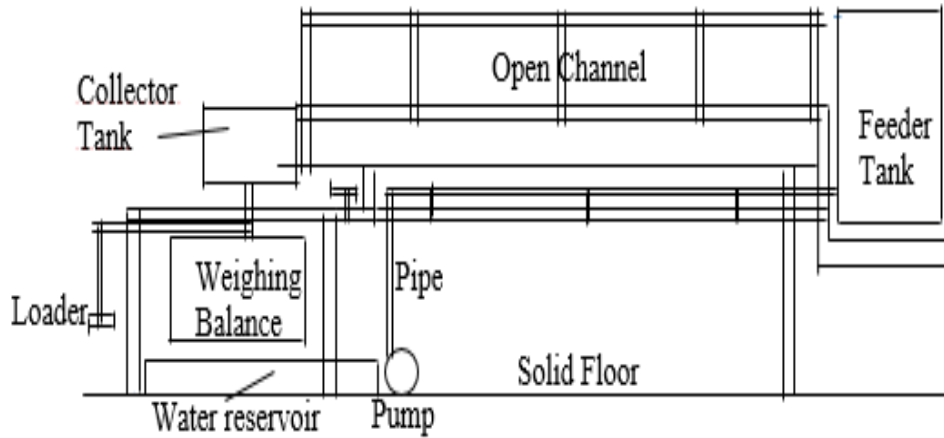
Fig. 4: Sample Constricted Flume and Hydraulic Jump

2. Experimental Test Set Up

The experiments were carried out on the horizontal open channel provided in the fluid mechanics/hydraulic laboratory of the Cross River University of Technology, Calabar- Nigeria. The channel dimensions are 0.075m (75mm) wide, and 0.15m (150mm) deep and 5.0m (5000mm) long. The channel is powered by a 1.13 KW (2 horse power) pump. It has an inlet feeder tank, braced smooth plexiglass perimeter base and walls and rubber packed sluice gate at the outlet. A constant head feeder tank provides the required discharge values and a pipe with a flow valve is used to regulate the

discharge. Measurement of water depths is performed using a mobile point gage attached to the channel. The flow velocity at any point is measured using stopwatch to time floating cocks (expanded polystyrene beads) and flow velocity at any point is then calculated as $\frac{\text{Distance travelled by cock}}{\text{Average time}}$. The flow

velocity value at any point is taken as the average of three measurements. In all the experiments, the weirs considered to be broad-crested since the water sheet flowing over the weir crest is less than twice the length of the weir (i.e. $y_c < 2 \times 0.114\text{m}$). The channel and whole setup scheme view is given in fig. 3 and 4 below.



Gravinometric Open Channel Apparatus

Fig. 3.: Genal Set Up View



Fig. 5: View of The Open Channel in the hydraulic laboratory of the Cross River University of Technology (CRUTECH), Calabar, Nigeria

3. General Experimental Procedures

1. General inspection of the channel condition.
2. Use 9 turns of the screw-jack near the channel outlet to adjust and provide a bed to a slope of 1:196.
3. Fill the reservoir full with water.
4. Remove the sluice gate, supply and run a constant water head through the channel.
5. Placed the three tablets 2.5m away from the channel outlet to create a broad crested weir 0.075m (75mm), 0.114m (114mm) long and 0.025m (25mm) high, with the weir tail corresponding to the 2.5m point of the channel.
6. Observe and measure relevant parameters.
7. Sandwich the 2 plexiglass slides on the opposite walls 2.5m away from the channel outlet to create a level-bedded constricted flume with a 0.50mm (50mm) throat.
8. Repeat step 6.
9. Adjust the flow valve to increase or decrease flow.
10. Repeat the experiment for at least 5 different flows.

B. Theory

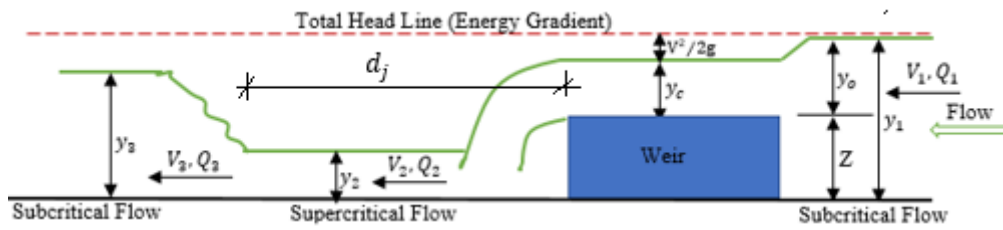


Fig. 6: Flow Over a Broad-Crested Weir

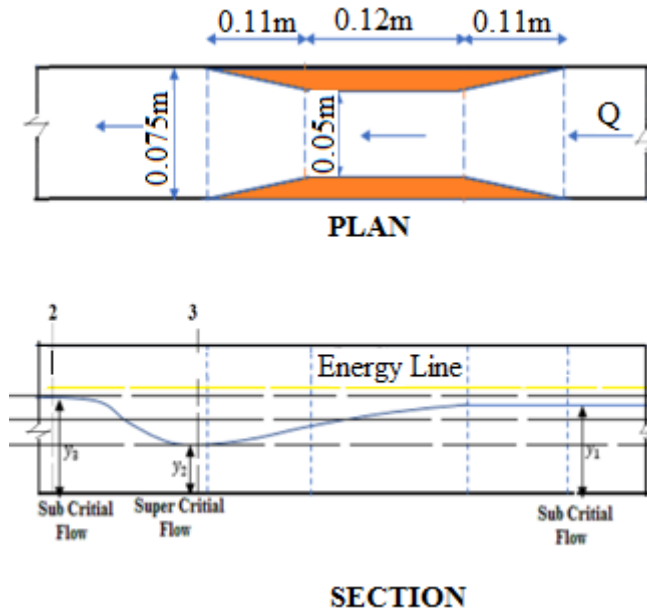


Fig. 7: Flow Through a Constricted Flume

Assuming no loss of head,

$$H_{\text{weir}} = y_1 = Z + y_o = Z + y_c + \frac{v^2}{2g} \quad (1)$$

Notation

b = channel width = 75mm = 0.075m

Z = weir depth (tablet thickness) in meters

v_1 = measured flow velocity upstream of weir

v_2, v_3 = measured pre/post-hydraulic flow velocities respectively

Q_1 = discharge upstream of weir

Q_2, Q_3 = measured pre/post-hydraulic flow discharges respectively

y_1 = measured water depth upstream of weir

y_2, y_3 = measured pre/post-hydraulic jump water depths (initial and sequent depths) respectively. Also known as conjugate depths

$$\text{Dimensionless depth ratio, } \frac{y_3}{y_2} = \frac{1}{2} \left(\sqrt{1 + 8F_{r2}^2} \right) - 1 \quad (2)$$

y'_i = pre/post-hydraulic jump alternate water depth

$$= \frac{2y_i}{-1 + \sqrt{1 + \frac{8gy_i^3}{(q)^2}}} \quad (3)$$

ΔE = The loss of energy head due to the occurrence of hydraulic jump

$$\Delta E = E_2 - E_3 = \left(y_2 + \frac{v_2^2}{2g} \right) - \left(y_3 + \frac{v_3^2}{2g} \right), \quad \text{for a rectangular open channel} = \frac{(y_3 - y_2)^3}{4y_3y_2} \quad (4)$$

where E_2 is pre-hydraulic jump Specific Energy of Supercritical flow, and, E_3 is post-hydraulic jump Specific Energy of Subcritical flow respectively. Alternatively,

$$\Delta E' = E'_2 - E'_3 = \left(y'_2 + \frac{q^2}{2gy_2^2} \right) - \left(y'_3 + \frac{q^2}{2gy_3^2} \right) \quad (5)$$

Froude number at any stream point = $\frac{V_i}{\sqrt{gy_i}}$

$$(6)$$

$$g = 9.806 \text{ m/s}^2$$

IV. RESULTS AND DISCUSSIONS

Table 1: General Experimental Data

Weir										
		Upstream of Weir			Upstream of Hydraulic Jump			Downstream of Hydraulic Jump		
1	2	3	4	5	6	7	8	9	10	11
Flow	Z (m)	y_1 (m)	v_1 (m/s)	$Q_1 \times 10^{-4}$ (m ³ /s)	y_2 (m)	v_2 (m/s)	$Q_2 \times 10^{-4}$ (m ³ /s)	y_3 (m)	v_3 (m/s)	$Q_3 \times 10^{-4}$ (m ³ /s)
1	0.075	0.097	0.067	4.87	0.007	0.93	4.89	0.032	0.21	4.90
2	0.075	0.097	0.080	5.67	0.008	0.94	5.66	0.034	0.22	5.66
3	0.075	0.097	0.082	5.96	0.008	0.99	5.94	0.036	0.22	5.91
4	0.075	0.097	0.083	6.04	0.009	0.90	6.06	0.034	0.24	6.12
5	0.075	0.097	0.087	6.32	0.009	0.94	6.35	0.036	0.24	6.53
6	0.075	0.099	0.088	6.55	0.009	0.97	6.53	0.037	0.23	6.40
Constricted Flume										
1		0.028	0.23	4.89	0.012	0.54	4.90	0.023	0.30	4.91
2		0.028	0.27	5.68	0.013	0.58	5.70	0.024	0.31	5.72
3		0.029	0.27	5.98	0.013	0.60	5.71	0.025	0.31	5.72
4		0.035	0.23	6.08	0.013	0.62	6.10	0.026	0.31	6.12
5		0.037	0.23	6.35	0.013	0.65	6.40	0.028	0.31	6.53
6		0.038	0.23	6.67	0.013	0.67	6.70	0.029	0.31	6.65

Column 5, 8 and 11 show that there is flow continuity since the discharge through any point of the channel is approximately equal.

Table 2: The Energy Head Loss Due to Hydraulic Jump

Weir										
Flow	Sequent/Conjugate Depths		Alternate Depths		Real			Alternative		
	y_2 (m)	y_3 (m)	y_2' (m)	y_3' (m)	E_2 (m)	E_3 (m)	ΔE (m)	E_2' (m)	E_3' (m)	$\Delta E'$ (m)
1	0.007	0.032	0.050	0.009	0.051	0.032	0.019	0.051	0.032	0.019
2	0.008	0.034	0.052	0.011	0.053	0.036	0.017	0.053	0.036	0.017
3	0.008	0.036	0.057	0.011	0.058	0.038	0.020	0.058	0.038	0.020
4	0.009	0.034	0.049	0.011	0.050	0.037	0.013	0.050	0.037	0.013
5	0.009	0.036	0.053	0.012	0.054	0.039	0.015	0.054	0.039	0.015
6	0.009	0.037	0.055	0.012	0.057	0.040	0.017	0.057	0.040	0.017
Constricted Flume										
1	0.012	0.023	0.023	0.012	0.026	0.026	0.00	0.026	0.026	0.00
2	0.013	0.024	0.026	0.026	0.030	0.029	0.001	0.030	0.029	0.001
3	0.013	0.025	0.026	0.026	0.031	0.030	0.001	0.031	0.030	0.001
4	0.013	0.026	0.029	0.029	0.033	0.034	-0.001	0.033	0.034	-0.001
5	0.013	0.028	0.031	0.031	0.035	0.036	-0.001	0.035	0.036	-0.001
6	0.013	0.029	0.033	0.033	0.035	0.035	0.00	0.035	0.035	0.000

For the weir, the energy loss range due to hydraulic jump was 0.013-0.020. While for the level-bedded constricted flume, the energy loss range due to hydraulic jump was -0.001 to 0.001 showing some energy gain with increase in rate of discharge through the flume. This implies that hydraulic jump resulting from a weir dissipates more energy than that caused by a level-bedded constricted flume.

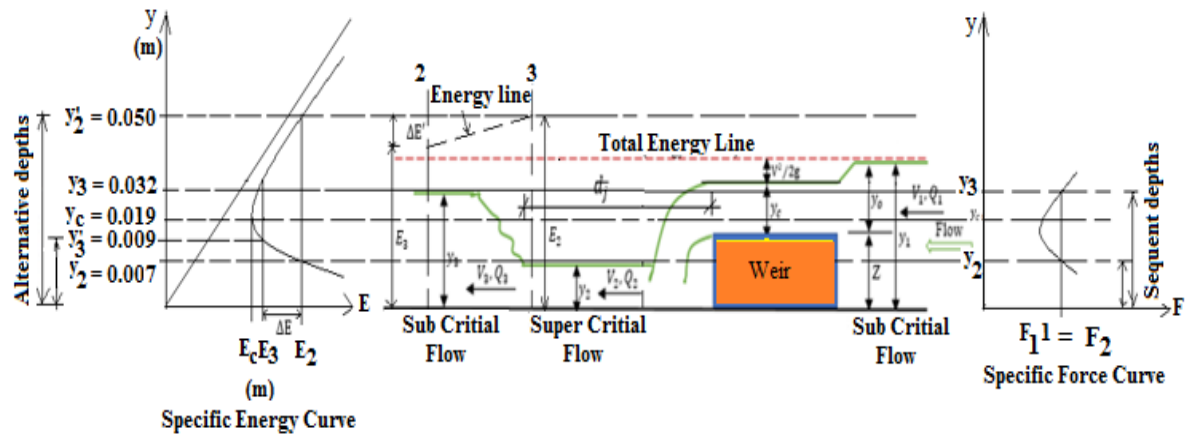


Fig. 8: Sample Specific Energy and Specific Force Curves for Flow 1 (Weir)

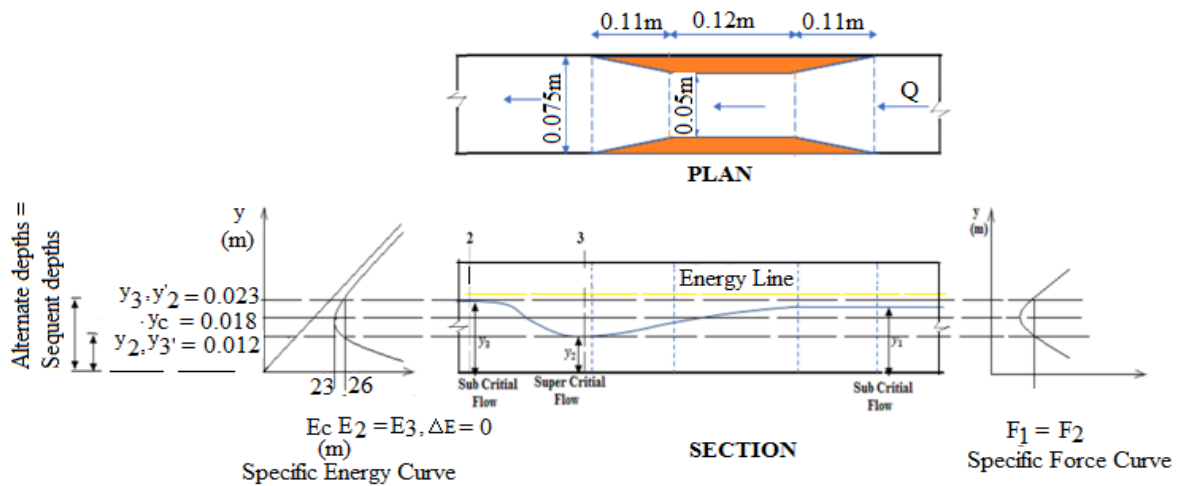


Fig. 9: Sample Specific Energy and Specific Force Curves for Flow1 (Constricted Flume)

Table 3: Froude Number

Weir						Boundaries	
Flow	F_{r1}	F_{r2}	F_{r3}	$\frac{y_3}{y_2}$	$\frac{v_3}{v_2}$		
1	0.068	3.56	0.37	4.26	4.26	Supercritical flow	$F_r=1-1.7$ undular jump
2	0.082	3.37	0.38	4.29	4.29		
3	0.084	3.56	0.37	4.52	4.52		
4	0.090	3.02	0.41	3.80	3.80	Critical flow	$F_r=1.7-2.5$ weak jump
5	0.089	3.17	0.40	4.01	4.01		
6	0.089	3.27	0.39	4.13	4.13		
Constricted Flume						Subcritical flow	$F_r=2.5-4.5$ oscillating jump 1.93
1	0.44	1.59	0.68	1.80	1.80		
2	0.52	1.64	0.64	1.87	1.87		
3	0.52	1.64	0.64	1.87	1.87		$F_r=4.5-9.0$ steady jump
4	0.40	1.75	0.61	2.03	2.03		
5	0.38	1.84	0.58	2.15	2.15		
6	0.38	1.93	0.56	2.27	2.27		$F_r > 9.0$ Strong jump

Upstream of weir, the Froude numbers range from 0.068 to 0.090 ($0.068 < F_{r1} < 0.09$), showing that the flows were subcritical. At the pre-hydraulic jump section, the Froude numbers range from 3.02 to 3.56 ($3.02 < F_{r2} < 3.56$), showing that the flows were supercritical and the jumps obtained were oscillating ones. The Froude numbers from the post-hydraulic jump section range from 0.37 to 0.41 ($0.37 < F_{r3} < 0.41$), also showing that the flows are subcritical.

Upstream of level-bedded constricted flume, the Froude numbers range from 0.038 to 0.052 ($0.038 < F_{r1} < 0.52$), showing that the flows were subcritical. At the pre-hydraulic jump section, the Froude numbers range from 1.59 to 1.93 ($1.59 < F_{r2} < 1.93$), showing that the flows were supercritical and the jumps obtained were weak ones. The Froude numbers from the post-hydraulic jump section range from 0.56 to 0.68 ($0.56 < F_{r3} < 0.68$), also showing that the flows are subcritical.

On the whole, the experiment results show that in an open channel hydraulic jump actually occurs only when there is flow continuity and when a flowing liquid transits from supercritical flow to subcritical one.

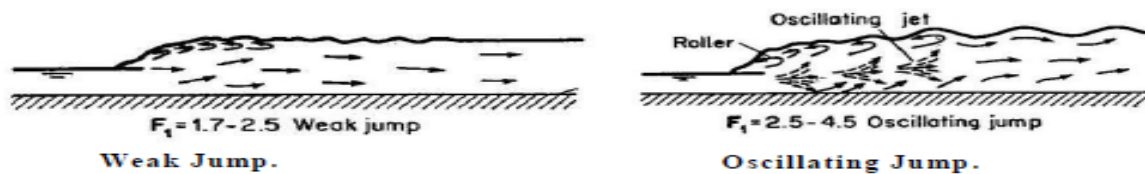


Fig. 10: Classification of Hydraulic Jump. Reprinted From “Experimental Study of Hydraulic Jump In Open Channel: Measurement And Comparison Of Theoretically Predicted Profile” by Lalnunluanga, Examination Roll No. M4MEC13-40 University Registration No. 113533 of 2010 – 11.

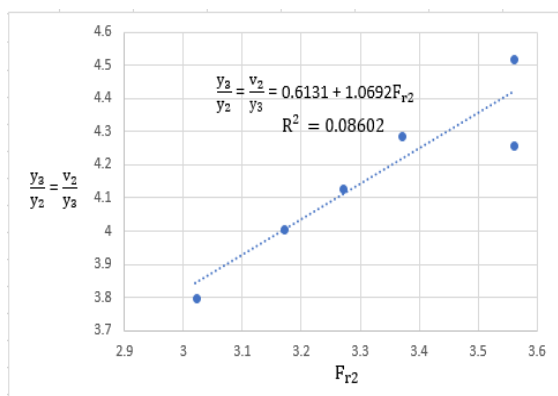


Fig. 11: Sequent Depth Ratio or Velocity Ratio vs. The Inflow Froude Number (Weir)

Fig. 11. shows that for the weir, the relationship between sequent depth ratio or velocity ratio is given as $\frac{y_3}{y_2} = \frac{v_2}{v_3}$ as $-5024 + 1.485F_{r2}$ with $R^2 = 0.9957$ showing that as the sequent depth ratio or velocity ratio increased, the inflow Froude number F_{r2} also increased.

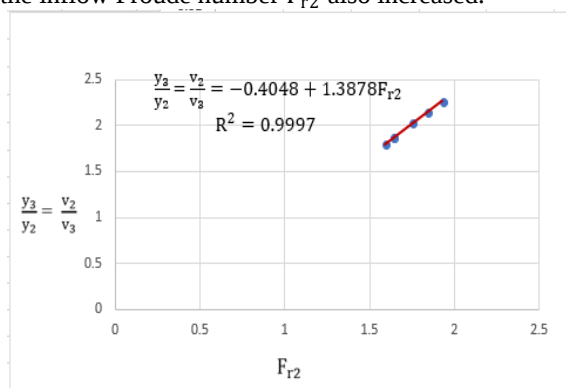


Fig. 12: Sequent Depth Ratio or Velocity Ratio vs. The Inflow Froude Number (Constricted Flume)

Fig. 12. also shows that for the constricted flume, the relationship between sequent depth ratio or velocity ratio is given as $\frac{y_3}{y_2} = \frac{v_2}{v_3} = -0.4048 + 1.3875F_{r2}$ with $R^2 = 0.9997$ showing that as the sequent depth ratio or velocity ratio increased, the inflow Froude number F_{r2} also increased.

V. CONCLUSIONS

Following the experiment, theory and analyses done, the following conclusions were derived:

1. For the weir, the energy loss due to hydraulic jump ranged from 0.013-0.020. Upstream of weir, the Froude numbers range from 0.068 to 0.090 ($0.068 < F_{r1} < 0.09$), showing that the flows were subcritical. At the pre-hydraulic jump section, the Froude numbers range from 3.02 to 3.56 ($3.02 < F_{r2} < 3.56$), showing that the flows were supercritical and the jumps obtained were oscillating ones. The Froude numbers from the post-hydraulic jump section range from 0.37 to 0.41 ($0.37 < F_{r3} < 0.41$), also showing that the flows are subcritical. The relationship between sequent depth ratio $\frac{y_3}{y_2}$ or velocity ratio $\frac{v_2}{v_3}$ is approximately $-5024 + 1.485F_{r2}$ with $R^2 = 0.9957$ showing that as the sequent depth ratio or velocity ratio increased, the inflow Froude number F_{r2} also increased.
2. For the level-bedded constricted flume, the energy loss due to hydraulic jump ranged from -0.001 to 0.001 showing some energy gain with increase in rate of discharge through the flume. Upstream of flume, the Froude numbers range from 0.038 to 0.052 ($0.038 < F_{r1} < 0.52$), showing that the flows were subcritical. At the pre-hydraulic jump section, the Froude numbers range from 1.59 to 1.93 ($1.59 < F_{r2} < 1.93$), showing that the flows were supercritical and the jumps obtained were weak ones. The Froude numbers from the post-hydraulic jump section range from 0.56 to 0.68 ($0.56 < F_{r3} < 0.68$), also showing that the flows are subcritical. The relationship between sequent depth ratio $\frac{y_3}{y_2}$ or velocity ratio $\frac{v_2}{v_3}$ is approximately $-0.4048 + 1.3878F_{r2}$ with $R^2 = 0.9997$ showing that as the sequent depth ratio or velocity ratio increased, the inflow Froude number F_{r2} also increased.

$<F_{r1}<0.52$), showing that the flows were subcritical. At the pre-hydraulic jump section, the Froude numbers range from 1.59 to 1.93 ($1.59 < F_{r2} < 1.93$), showing that the flows were supercritical and the jumps obtained were weak ones. The Froude numbers from the post-hydraulic jump section range from 0.56 to 0.68 ($0.56 < F_{r3} < 0.68$), also showing that the flows are subcritical. The relationship between sequent depth ratio $\frac{y_3}{y_2}$ or velocity ratio $\frac{v_2}{v_3}$ is approximately $-0.4048 + 1.3875F_{r2}$ with $R^2 = 0.9997$ also showing that as the sequent depth ratio or velocity ratio increased, the inflow Froude number F_{r2} also increased.

3. On the whole, the experiment has proven that in an open channel hydraulic jump actually occurs only when there is flow continuity and when a flowing liquid transits from supercritical flow to subcritical one. Hydraulic jump resulting from a weir dissipates more energy than that caused by a level-bedded constricted flume. The inflow Froude number F_r of a hydraulic jump is a function of sequent depth ratio or velocity ratio of the post and pre-hydraulic jump sections, irrespective of the jump causative agent.

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