



Stochastic Pricing of European Options Using the Black-scholes Model on the Analysis of Share Prices for Capital Markets

Obeten, B.O ^{a*}, Chims, B.E ^b, Nwaigwe, E ^c, Amadi, I.U ^c
and Davies, I. ^d

^a Department of Statistics, University of Cross Rivers State, Nigeria.

^b Department of Statistics, Ken-Sarowiwa Polytechnic, Bori, Port Harcourt, Nigeria.

^c Department of Mathematics & Statistics, Captain Elechi Amadi Polytechnics, Port Harcourt, Nigeria.

^d Department of Mathematics, Rivers State University Nkpolu Oroworokwo, Port Harcourt, Nigeria.

Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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ABSTRACT

Financial instruments used in the capital market to finance long-term investments are stocks and shares, company bonds and government bonds. This involves the issue and market of shares, bonds and debentures using the services of brokers, dealers and underwriters. The capital market provides a means through which this is made possible. However, the paucity of long-term capital

*Corresponding author: Email: brightobeten86@gmail.com;

has posed the greatest challenge. This study is about option pricing and some of its dynamics in financial markets via valuation using the Black-Scholes (BS) model, as it explores the changes of option values as a function of security and time. The study is based on Stochastic Differential Equations and the Black-Scholes Model. The findings revealed that the Black-Scholes model of European options on the share price of Fidelity, Access and their future merged banks, which gave closed-form prices of Call and Put option prices with variations of maturity dates, average share prices, as well as their respective standard deviations affecting real-life changes for capital markets. From the share price analysis, the growth rates of each bank were considered, where Fidelity Bank had the largest growth rates, as this is informative to investors or management of the banks in terms of decision making. This paper offers a reflective consequence for future studies of option prices. However, the current study is on the European options case. Another study could be considered in the case of multiple options in one portfolio of investments.

Keywords: Capital market; stochastic pricing; Black-Scholes (BS) model; share price; financial instruments.

1. INTRODUCTION

“The capital market has been identified as an institution that contributes to the economic growth and development of emerging economies like Nigeria. The capital market is made up of financial institutions which deal in long-term loans for investment. They therefore bring long-term lenders and borrowers together. The main role of the capital market is to raise finance for different institutions. Raising finance will go through issuing a variety of securities. Short-term or working capital is channelled through borrowing from the money market. This happens by issuing various securities such as bills, promissory notes, etc”. (Algaeed, 2021; Wójtowicz & Czupryn, 2023). “Loans given are usually for more than two years. Institutions that operate in this market include: insurance companies, issuing Houses, Development Banks, Investment Banks, Investment Trusts, Building Societies or Mortgage Banks, Finance Corporations, Savings Banks and Stock Exchanges. Financial instruments used in the capital market to finance long-term investments are stocks and shares, company bonds and government bonds. Capital market is a profoundly particular and coordinated monetary market and a fundamental instrument of financial development, inferable to its capacity to attract and assemble investment funds and speculation” (Toby & Dibiah, 2021). “The capital market can be divided into the primary and the secondary market. The primary market deals with the buying and selling of new securities. The secondary market is the market that deals with the buying and selling of already existing (secondhand) securities. It is dominated by the Stock Exchange. In the narrowest sense, the capital market involves the problems and

prospects of equity investment. This involves the issue and market of shares, bonds and debentures using the services of brokers, dealers and underwriters. The capital market provides a means through which this is made possible. However, the paucity of long-term capital has posed the greatest challenge. In capital investments, money is being invested in a business such that the return rates will be appropriately utilised to cover day-to-day trading activities expenses. An investor may need additional capital assets in order to improve its trading events. So capital investments are established on the basis of: to acquire additional capital assets for expansion, which enables the business to increase in unit production, create new ideas on products or even add value to the business and to explore on technological advancements in order to increase efficiency and reduce costs and to replace worn-out assets. In the absence of capital investments, the trading business will definitely have a hard time getting off the ground” (Azor et al, 2023).

However, since the beginning of the stock market, investors have been trying to gain an edge. The number-crunchers out there have long sought out a mathematical model which could predict price fluctuation. If they could somehow figure out the secret formula, they could become rich beyond their wildest dreams. A team of economists, Fischer Black, Myron Scholes, and Robert C. Merton, tried to do just that, and they came up with a powerful mathematical model for financial markets that contain derivative instruments. Lacking a creative name, this model became known as the Black-Scholes-Merton model. From this larger model, smaller models and equations were made based on the same assumptions. After years of developing the

model, Robert Merton is attributed with first mentioning the "Black-Scholes options pricing model" in 1973. This theoretical model could help options market-makers properly price options on all types of financial instruments. Their work was so ground-breaking that 24 years later, in 1997, Robert C. Merton and Myron Scholes won the Nobel Memorial Prize in Economic Studies for their work. Therefore, Black-Scholes is a pricing model used to determine the fair price or theoretical value for a call or a put option based on six variables such as volatility, type of option, underlying stock price, time, strike price, and risk-free rate. The quantum of speculation is more in the case of stock market derivatives, and hence, proper pricing of options eliminates the opportunity for any arbitrage. There are two important models for option pricing – the Binomial Model and the Black-Scholes Model. The model is used to determine the price of a European call option, which simply means that the option can only be exercised on the expiration date. The Black-Scholes model is used to calculate the theoretical price of European put and call options, where an option is a contract for the right to buy and sell shares at a later date or within a certain period at a particular price. Therefore, they assumed some features of the financial market, including European-style options. The model supposes European-style options. Those can only be exercised on the expiration date. With American-style options, it is possible to exercise the option at any time during the life of the option. Efficient markets: It is assumed that the stock's behaviour is like a random walk. Meaning, at any given moment in time, the price of the underlying stock can go up or down. The future stock price is independent of the past. The market movements cannot be predicted.

The Black-Scholes formula is a mathematical model to calculate the price of put and call options. Since put and call options are distinctly different, there are two formulas which account for each option. Call options give the option holder the right to buy the underlying stock for an agreed-upon price anytime between today and upon expiration. Traders who believe the underlying stock will go up over time buy these call options in the hopes of making money. On the flip side, put options give the option holder the right to sell the underlying stock for an agreed-upon price anytime between today and upon expiration. Traders who think a stock is going to go down can buy these put options in

the hopes of making money if the stock goes down.

However, the mathematical modelling of stock options has called for great concern to all scholars, as it is expected that this time the empirical approach will give answers to many problems. For instance, Fadugba & Nwozo (2013) had an empirical study on the valuation of options. They applied CN FDM and concluded that CN is unconditionally stable, convergent and more accurate when pricing European options. Osu et al. (2009) studied the stability analysis of a stochastic model of stock market price. They applied the method of Black- Scholes analysis using Crank-Nicolson as a numerical scheme. In their research, stock prices were stable, and increase rate of the stock shares was also determined. Tangman et al. (2008) examined "High Order Compact (HOC) Schemes for quasilinear parabolic PDEs to discretise the BS PDE for numerical pricing of European and American options". Along the line, Dremkova & Ehrhardt (2011) presented "compact finite difference schemes to solve nonlinear BS Equations for American options with a nonlinear volatility function. In that research, it was discovered that the compact scheme cannot be applied effectively to American options; the study finally used a fixed domain transformation to obtain its results". During the same period, Song & Wang (2013) used "symbolic calculator software to provide a numerical solution applying the implicit scheme of FDM. This research combined the time fractional BS equation with the conditions sufficient for the normal put options". Some years later, Uddin et al. (2015) presented "results for European call and put options using FDM and Finite Element Method (FEM)". In a recent study, Zhang et al. (2006) studied "the tempered fractional derivative to price a European double-knock barrier option. They analysed characteristics of the three fractional BS models through resemblance with the classical BS model". Much earlier, Cortes et al. (2005) included "a vital aspect of errors into the numerical solution. They introduced the Mellin transformation and proposed that the errors of composite Simpson's rule or Euler's method can be avoided while pricing the BS equation in the real world of financial derivatives". Company et al. (2006) also used "the Mellin transform and a delta-defining sequence of the involved established Dirac delta function to assign a numerical solution". Other empirical studies on numerical solution considered different aspects, for instance,

Company et al. (2008) employed “the semi-discretisation technique to deal with the issues arising as a result of a nonlinear case of interest, modelling option pricing with transaction costs. The nonlinear BS models have gotten the attention of researchers because of the rational assumptions, including transaction costs, high volatility, illiquid markets and large investors' preferences, can also be included”. Abkudinova & Ehrhardt (2008) examined that “the CN and the R3C scheme are the most accurate techniques to price the European call option. This study addresses different volatility problems in stock price, option price and its derivatives”. Rao (2016) also applied “a numerical scheme to the generalised Black-Scholes models for European call options. The outcome showed that the second-order accuracy in time and the third-order accuracy in space were obtained”. Amadi et al. (2020) worked on “the Black-Scholes partial differential equation on stock market prices. They applied the B.S analytical formula and Crank-Nicolson numerical method. It was observed that Black-Scholes and Crank-Nicolson are impossible to differentiate, but in terms of precision, Black-Scholes analytical values were found to be more adequate”. Furthermore, Shim & Kim (2016) focused on “the Black-Scholes terminal value problem and provided its solutions through the Laplace transform. This study claimed that the proposed method is simpler than the existing methods” Amadi et al. (2024). Amadi et al. (2024) studied “the perception of European option, which is geared towards the valuation of financial assets, the application of share prices of Fidelity and Access banks, which gave closed-form prices of call options. The explicit price on the variations of maturity days is found accordingly”.

In the same vein, Babasola et al. (2008) analysed “the BS formula for the valuation of European options; Hermite polynomials were applied. They concluded that the BS formula can easily be achieved devoid of the use of a partial differential equation”. In another study of BS, Shin & Kim (2016) considered “the BS terminal value problem and observed that their proposed method is better, simpler than the previous methods”. In the work of Rodrigo et al. (2006), “time-varying factors were incorporated in the explicit formula for different aspects of options with the aim of providing an exact solution for dividend-paying equity options. In considering the stability of stock market price of the stochastic model”; Osu et al. (2019) applied the Crank-Nicolson numerical scheme to the BS

model. The results showed stock prices being stable, and its increasing rate of stock shares was obtained. However, Nwobi et al (2019) studied “the Black-Scholes model because of its bias in mispricing options. They established a new technique of assessing pricing effects on the premise of reducing pricing bias”.

However, reviews show that Black-Scholes have copious applications as seen in the literature. This present paper is aimed at pricing European options of Share prices Fidelity and Access banks, future merging and determining their growth-rates for capital market investments as this will add values in this dynamic area of mathematical finance.

The major issues for investors or owners of corporations are the inability to make appropriate decisions when using the Black-Scholes model of option pricing for applications. These issues may have arisen due to the formulation of the problem or inability to add more models to the study, or inability to understand the analytical solutions to accurately interpret in the real world market. These discrepancies may lead to models not predicting closed-form solutions for European options for proper decision making, which may not be in the best interest of option traders or investors or management of Fidelity and Access banks. In order to tackle the above problem, we impose European options to value shares for Call and Put options and to determine their various growth rates to realistically assess share prices for capital markets.

2. MATERIALS AND METHODS

In this Section, we present some rudimentary preliminaries touching the dynamics of the study, hence we have as following:

2.1 Stochastic Processes

It can also be seen as a statistical event that evolves over time in accordance with probabilistic laws. Mathematically, a stochastic process may be defined as a collection of random variables which are ordered in time and defined at a set of time points, which may be continuous or discrete.

2.2 Stochastic Differential Equations

Here, consider a market where the underlying asset price v , $0 \leq t \leq T$ on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ is governed by the following stochastic differential equation:

$$dS(t) = \alpha S(t)dt + \sigma dw(t). \quad 0 < v < \infty. \quad (2.1)$$

Theorem 2.1: (Ito's formula) Let $(\Omega, \beta, \alpha, F(\beta))$ be a filtered probability space $X = \{X, t \geq 0\}$ be an adaptive stochastic process on $(\Omega, \beta, \alpha, F(\beta))$ possessing a quadratic variation (X) with SDE defined as:

$$dX(t) = g(t, X(t))dt + f(t, X(t))dW(t)$$

$$t \in \mathfrak{R} \text{ and for } u = u(t, X(t)) \in C^{1 \times 2}(\Pi \times \square)$$

$$du(t, X(t)) = \left\{ \frac{\partial u}{\partial t} + g \frac{\partial u}{\partial x} + \frac{1}{2} f^2 \frac{\partial^2 u}{\partial x^2} \right\} d\tau + f \frac{\partial u}{\partial x} dW(t)$$

Adopting Theorem 2.1 comfortably solves the SDE in (3/1) with a given solution below:

$$S(t) = S_0 \exp \left\{ \sigma dW(t) + \left(\alpha - \frac{1}{2} \sigma^2 \right) t \right\}, \forall t \in [0, 1] \quad (2.2)$$

2.3 The Black-Scholes Model

The seminar paper on option pricing by Fisher Black and Myron Scholes in 1973 (Black & Scholes 1973) has had a great impact on contemporary research in the Mathematics of finance. This model is commonly used in financial modelling. The Black-Scholes model is made up of on seven assumptions:

- The asset price has characteristics of a Brownian motion with μ and σ as constants.
- The transaction costs or taxes are not allowed.
- The entire securities are absolutely divisible.
- Dividend is not permitted during the period of the derivatives.
- Unacceptable of riskless arbitrage opportunities.
- The security trading is constant.
- The option is exercised at the time of expiry for both call and put options.

In mathematical finance, an arbitrage arguments show that any derivative $V(S, t)$ written on v must satisfy the partial differential equation of the form of option pricing; hence, we have the following:

$$\frac{\partial V(S, t)}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V(S, t)}{\partial S^2} + rS \frac{\partial V(S, t)}{\partial S} - rV(S, t) = 0 \quad (2.3)$$

Where r represents interest rate, σ represents volatility of the underlying assets and t represents time of maturity.

With boundary conditions:

$$V(S, t) \rightarrow \infty \text{ as } S \rightarrow \infty \text{ on } [0, T). \quad (2.4)$$

$$V(S, t) \rightarrow 0 \text{ as } S \rightarrow 0 \text{ on } [0, T). \quad (2.5)$$

And the final time condition given by:

$$V(S_T, T) = (S_T - k)^+ = f(S_T) \text{ on } [0, \infty]. \quad (2.6)$$

Equation (2.3) is the value of the asset is worthless when the stock price is zero, Amadi et al., (2024). The details of the above option model can be expressly be found in the following books: Black-Scholes (1973), Hull (2012), etc.

To eliminate the price process in (2.3- 2.6) slightly gives the Black-Scholes analytic formula for the prices of European call options is given as follows

$$\left. \begin{aligned} C &= SN(d_1) - Ke^{-r}N(d_2) \\ d_1 &= \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}} \\ d_2 &= d_1 - \sigma\sqrt{\tau} \end{aligned} \right\} \quad (2.7)$$

where C is Price of a call option, S is price of underlying asset, K is the strike price, r is the riskless rate, T is time to maturity, σ^2 is variance of the underlying asset, σ is the standard deviation of the (generally referred to as volatility) underlying asset, and N is the cumulative normal distribution.

Similarly Black-Scholes analytic formula for the prices of European Putl option is given as follows

$$\left. \begin{aligned} P &= SN(d_1) - Ke^{-r}N(d_2) \\ d_1 &= \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}} \\ d_2 &= d_1 - \sigma\sqrt{\tau} \end{aligned} \right\} \quad (2.8)$$

where P is the price of a put option and the meaning of other parameters remain the same as in (2.7) Hull (2003).

However, the overall expected growth rates for Fidelity, Access banks and their future merging is defined in a risk-neutral world as follows:

$$Growth(GR) = \ln \left(\frac{\text{Max. share price}}{\text{Min. share price}} \right) \times 100 \text{ percent}$$

3. RESULTS AND DISCUSSION

This Section presents analysed results whose methods are stated in the above section. Hence, we have the following parameter values:

Fidelity and Access bank : $r = 0.03$, $\sigma = 0.25$, $k = 450$ and $t = 1$, *Merged Banks* : $k = 700$ which were implemented using MATLAB programming software:

Table 1. The value of the Share price of Fidelity Bank, PLC, according to trading days for European Call Option

Initial share price (S_0)	The value of share prices when time $t = 6$	The value of share prices when time $t = 12$	Average Value of share prices	STD
415	100	140	120	28.2843
62	0.016	0.445	0.2305	0.3033
138	1.653	8.00	4.8265	4.4880
61	0.014	0.417	0.2155	0.2850
121	0.852	5.232	3.0420	3.0971
81	0.088	1.268	0.6780	0.8344
139	1.713	8.188	4.9505	4.5785
80	0.081	1.209	0.6450	0.7976
384	81.09	119	100.0450	26.8064

Table 2. The value of Share price of Fidelity Bank, PLC According to trading days for European Put Option

Initial share price (S_0)	The value of share prices when time $t = 6$	The value of share prices when time $t = 12$	Average Value of share prices	STD
415	407	413	410	4.2
62	52	53	53	0.7
138	130	133	132	2.1
61	51	52	52	0.7
121	113	116	115	2.1
81	72	73	73	0.7
139	131	134	133	2.1
80	71	72	72	0.7
384	376	382	379	4.2

Table 3. The value of Share price of Access Bank, PLC According to trading days for European Call Option

Initial share price (S_0)	The value of share prices when time $t = 6$	The value of share prices when time $t = 12$	Average Value of share prices	STD
410	97	137	117	28.28
80	0.08	1.21	0.64	0.80
126	1.05	5.98	3.52	3.49
79	0.075	1.15	0.61	0.76
98	0.271	2.54	1.41	1.60

Initial share price (S_0)	The value of share prices when time $t = 6$	The value of share prices when time $t = 12$	Average Value of share prices	STD
92	0.188	2.028	1.11	3.30
127	1.093	6.13	3.65	3.56
91	0.176	1.95	1.06	1.25
378	77.57	114.1	96	25.83

Table 4. The value of Share price of Access Bank, PLC According to trading days for European Put Option

Initial share price (S_0)	The value of share prices when time $t = 6$	The value of share prices when time $t = 12$	Average Value of share prices	STD
410	402	408	405	4.2
80	71	72	72	0.7
126	118	121	110	2.1
79	70	71	71	0.7
98	90	92	91	1.4
92	84	85	85	0.7
127	119	122	121	2.1
91	83	84	84	0.7
378	370	376	373	4.2

Tables 1-4 show the interpretations of Call and Put options with different maturity dates in terms of investment plans. Different maturity dates can give investors or banks more flexibility to adjust positions based on changing market conditions. For example, if the banks are bullish on a stock in the long-term but bearish in the short-term, they could buy a Call option with a long-term maturity date while simultaneously selling a Call option with a shorter term. Also, options with different maturity dates can be used to hedge against potential losses.

In columns 4 and 5 of Table 6 are the values of averages and standard deviations, respectively. The average share price can be used to determine whether a stock is overvalued or undervalued, which can help investors make decisions about when to buy or sell. Looking at the average share prices of Fidelity and Access banks over time can help investors identify patterns and trends that may be useful for making investment decisions. The average share price can be compared with different stocks or sectors, which can be helpful for diversifying portfolios.

Share price standard deviation is a measure of volatility or the extent to which a data set varies

from its mean value. In the context of Call and Put options with different maturity dates, standard deviation tells how much the price of the options is likely to fluctuate over time. A higher standard deviation indicates that the options are more volatile and may be riskier to hold. A low standard deviation indicates that the options are more stable and may be less risky. Standard deviation can be an important consideration for Fidelity and Access banks who are trying to manage their risk to make predictions about future price movements; see Tables 4-6, column 5.

Tables 5 and 6 show the value of Fidelity and Access Bank's share prices as they merge in future. It can be seen from the variations of maturity days that an increase in the maturity days increases the value of call option prices. Carefully looking at the call option prices as they merge, one will understand that it is more profitable for the two banks to merge because the value of their assets will increase tremendously, as seen above. This remark is encouraging in every investment because it is profit maximising, which will guide the management of banks the way of making decisions based on the levels of their investments.

Table 5. The value of the Share price of Fidelity-Access Merged according to trading days for European Call Option

Initial share price (S_0)	The value of share prices when time $t = 6$	The value of share prices when time $t = 12$	Average Value of share prices	STD
825	449	491	470	29.70
142	1.89	8.76	5.33	4.86
264	24.93	50	37.47	17.73
140	1.77	8	4.89	4.41
219	12.51	31	21.76	13.07
173	4.72	16	10.36	7.98
266	25.60	51	38.30	17.96
171	4.48	15	9.74	7.44
762	389	432	411	30.41

Table 6. The value of the Share price of Fidelity-Access Merged according to trading days for European Put Option

Initial share price (S_0)	The value of share prices when time $t = 6$	The value of share prices when time $t = 12$	Average Value of share prices	STD
825	817	823	820	4.2
142	134	137	136	2.1
264	256	261	259	3.5
140	132	135	134	2.1
219	211	216	214	3.5
173	165	169	167	2.8
266	258	263	261	3.5
171	163	167	165	2.8
762	754	760	757	4.2

Table 7. The share price growth-rates of Fidelity, Access banks and their future merging

Fidelity Bank ,PLC	Access Bank,PLC	Future Merged
190	163	176
80	45	62
81	46	63
68	22	45
40	6.0	24
53	32	43
55	33	44
157	142	149

Table 7 represents some levels of growth rates from Fidelity, Access and their future merged banks. As can be seen that a higher growth rate means that a share price has been increasing at a faster rate over a certain period of time. Shares in this table with higher growth rates may be attractive to investors because they have the potential to generate higher returns. Whereas a smaller growth rate means that a share price has been increasing at a slower rate over a certain period of time. Shares with smaller growth rates may be less attractive to investors because they

have less potential for generating high returns. In all, Fidelity Bank has the largest growth rates and profit index in terms of returns.

4. CONCLUSION

This paper studied the framework of the Black-Scholes model of European options on share price of Fidelity, Access and their future merged banks, which gave closed form prices of Call and Put option prices with variations of maturity dates, average share prices as well as their

respective standard deviations affecting real life changes for capital markets. From the share price analysis, the growth rates of each bank were considered, where Fidelity Bank had the largest growth rates, as this is informative in terms of decision making.

However, the current study is on European options case. Another study could be considered in the case of multi-options in one portfolio of investments.

DISCLAIMER (ARTIFICIAL INTELLIGENCE)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of this manuscript.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

- Algaeed, A. H. (2021). Capital market development and economic growth: An ARDL approach for Saudi Arabia, 1985-2018. *Business Economics and Management (JBEM)*, 22(2), 388-409.
- Amadi, I. U., Nwaigwe, E. & Azor P. A. (2020). Mathematical Analysis of Black-Scholes Partial Differential Equation on Stock prices. *African Journal of Mathematics and Statistics studies*, Volume 3, Issue 4, 1-11.
- Amadi, I.U., Nmerukini, A, Ogbogolgo, P.C., Aboko, S.I & Udoeye, A. (2024). The Analysis of Black-Scholes of Option pricing with Time-Varying parameters on Share prices for Capital Market. *Applied Mathematics and Information Sciences an International Journal*. 18(3),673-678.
- Ankudinova, J. and Ehrhardt, M. (2008). On the Numerical Solution of Nonlinear Black-Scholes Equations. *Computer Application*, 56, 799-812.
- Azor, P. A., Emomotimi, A. T., & Amadi, I. U. (2023). Stochastic Analysis for Assessment of Asset Value Changes in Oando, PLC. *Asian Journal of Pure and Applied Mathematics*, 345-356.
- Babasola, O. L., Irakoze, I. & Onoja, A. A. (2008). Valuation of European options within the Black-Scholes framework using the Hermite Polynomial. *Journal of scientific and Engineering Research*, 5(1), 213-221.
- Black, F. & Scholes, M. (1973). The pricing of options and corporate liabilities. *The journal of political Economy*, 81,637-654.
- Company, R., A. Gonzalez & L. Jodar, (2006). Numerical solution of modified Black-Scholes equation pricing stock options with discrete dividend. *Math Comput. Modell.*, 44: 1058-1068. DOI: 10/1016/j.mcm.2006.03.009.
- Company, R., A. Navarro, J. R. Pintos & E. Ponsoda, (2008). Numerical solution of linear Black-Scholes option pricing equations. *Comput. Math. Applic.*, 56: 813-821. DOI: 10/1016/j.camwa.2008.02.010.
- Cortes, J., L. Jodar, R. Sala & P. Sevilla-Peris, (2005). Exact and numerical solution of Black-Scholes matrix equation. *Applied Math. Comput.*, 160: 607-613. DOI: 10.1111/j.1475-6803.1996.tb00592.
- Dremkova, E. & Ehrhardt, M. (2011). A High-Order Compact Method for Nonlinear Black-Scholes Option Pricing Equations of American Options. *International Journal of Computer/Mathematics*, 88, 2782-2797.
- Fadugba, S.E. & Nwozo, C.R. (2013). Crank-Nicolson Finite Difference Method for Valuation of Options. *The pacific Journal of Science and Technology*, 14(2).
- Hull, J. C. (2012). *Option, Futures and other Derivatives* (eighth Edition). Prentice Hall, Boston Columbus Indianapolis New York San Francisco.
- Hull, J.C. (2003). *Options, Futures and other Derivatives*, (5 Edition). Prentice Hall International, London.
- Nwobi, F.N., Annorzie, M.N. & Amadi, I.U. (2019). Crank-Nicolson finite difference method in valuation of options. *Communications in Mathematical Finance*, Vol,8, No1, 93-122.
- Osu, B.O. Okoroafor, A .C & Olunkwa, C. (2009). Stability analysis of stochastic model of stock market price. *African Journal of Mathematics and Computer Science Research*, 2(6), 98-103.
- Osu, B.O., Emenyonu, S.C., Ogbogbo, C.P. & Olunkwa, C. (2019). Markov Models on Share Price Movements in Nigeria Stock Market Capitalization, *Applied Mathematics and Information Sciences an International Journal*, N0 2,1-9.
- Rao, S. C. S., (2016). High-order numerical method for generalized Black-Scholes model- proc. *Comput. Sci.*, 80: 1765. 1776 DOI:10.1016/j.procs.2016.05.441.

- Rodrigo, M. R., & R. S. Mamon (2006). An alternative approach to solving the Black Scholes equation with time-varying parameters. *Applied Math. Lett.*, 19: 398-402. DOI:10.1016/j.aml.2005.06.012.
- Shin, B. & H. Kim, (2016). The solution of Black-Scholes terminal value problem by means of Laplace transform. *Global Journal Pure Applied Mathematics*, 12: 4153 – 4158.
- Song, L. and Wang, W. (2013). Solution of the fractional Black-Scholes option pricing model by finite difference method. *Abs Applied Analysis*, 1-11.
- Tangman, D., Gopaul, A. & Bhuruth, M. (2008). Numerical pricing of options using high-order compact finite difference schemes. *Journal of Computer/Applied Mathematics*, 218, 270-280.
- Toby, A. J., & Dibiah, S. (2021). Capital market development and economic growth in Nigeria. *American International Journal of Economics and Finance Research*, 3(1), 16-38.
- Uddin, M.K.S., Siddiki, M.N.A.A. & Hossain, M.A. (2015). Numerical Solution Black-Scholes Model: A comparative overview. *Journal of Statistics and Mathematical Sciences*, 1(1), 1-7.
- Wójtowicz, Ł., & Czupryn, R. (2023). The impact of the capital market on economic growth in Luxembourg. *Central European Review of Economics & Finance*, 44(3), 175-185.
- Zhang, H., Liu, F., Turnar, I. & Chen, S. (2006). The Numerical Simulation of the tempered Fractional Black-Scholes Equation for European Double Barrier Option. *Applied Mathematics Modelling*, 40, 5819-5834.

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